Tests of the Conserved Vector Current and Partially Conserved Axial-Vector Current Hypotheses in High-Energy Neutrino Reactions*

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The following theorem is proved: Consider the high-energy neutrino reaction $\nu+\alpha\rightarrow l+\beta$, with α a nucleon or nucleus, l a lepton (e or μ) and β a system of strongly interacting particles. Suppose that the mass of α and the invariant mass of β are not equal, and that the lepton mass is neglected. Then when the lepton emerges with its momentum parallel to that of the neutrino, the squared matrix element, averaged over lepton spin, depends only on the *divergences* of the vector and the axial-vector currents. Tests of the conserved vector current and the partially conserved axial-vector current hypotheses, based on the theorem, are proposed.

I. INTRODUCTION

THERE is a characteristic property of neutrino reactions at high energy which makes possible new tests of the conserved vector current[1] (CVC) and the partially conserved axial-vector current[2] (PCAC) hypotheses. Consider the reaction $\nu+\alpha\rightarrow l+\beta$, where α is a nucleon or nucleus, l is a muon or electron, and $\beta=\beta_1+\cdots+\beta_n$ is a system of strongly interacting particles. Let the four-momenta of ν, α, l, and β be, respectively, k_1, p_1, k_2, and p_2, and let the leptonic momentum transfer be $k=k_1-k_2=p_2-p_1$. We denote by M_α the mass of α, by m_l the mass of the lepton l, and by W the invariant mass of the system β. We take the neutrino mass to be zero.

Theorem 1. Suppose that $W\neq M_\alpha$ and that m_l is neglected. Consider the configuration in which the final lepton emerges with its momentum parallel to that of the incident neutrino. (We call this the *parallel configuration*.[3]) Then the squared matrix element for $\nu+\alpha\rightarrow l+\beta$, averaged over lepton spin, depends only on the divergences of the vector and the axial-vector currents.

Proof: The matrix element is

$$\mathfrak{M}=2^{-1/2}i\bar{u}_l\gamma_\lambda(1+\gamma_5)u_\nu\langle\beta|\mathcal{J}_\lambda{}^V+\mathcal{J}_\lambda{}^A|\alpha\rangle. \quad (1)$$

Squaring and averaging over lepton spin gives[4]

$$\langle|\mathfrak{M}|^2\rangle=\langle\beta|\mathcal{J}_\lambda{}^V+\mathcal{J}_\lambda{}^A|\alpha\rangle\langle\beta|\mathcal{J}_\sigma{}^V+\mathcal{J}_\sigma{}^A|\alpha\rangle^\star T_{\lambda\sigma}, \quad (2)$$

with

$$T_{\lambda\sigma}=k_{1\lambda}k_{2\sigma}+k_{1\sigma}k_{2\lambda}-k_1\cdot k_2\delta_{\lambda\sigma}+\epsilon_{\lambda\sigma\gamma\delta}k_{1\gamma}k_{2\delta}. \quad (3)$$

When m_l is neglected, k_1 and k_2 are null vectors. In the parallel configuration they are proportional. If $W\neq M_\alpha$, k_0 is nonzero,[5] and we may write

$$k_1=k_{10}k_0{}^{-1}k, \quad k_2=k_{20}k_0{}^{-1}k. \quad (4)$$

Thus,

$$T_{\lambda\sigma}=2k_{10}k_{20}k_0{}^{-2}k_\lambda k_\sigma. \quad (5)$$

Since $\langle\beta|\partial\mathcal{J}_\lambda/\partial x_\lambda|\alpha\rangle=-ik_\lambda\langle\beta|\mathcal{J}_\lambda|\alpha\rangle$, we find that

$$\langle|\mathfrak{M}|^2\rangle=2k_{10}k_{20}k_0{}^{-2}|\langle\beta|\partial(\mathcal{J}_\lambda{}^V+\mathcal{J}_\lambda{}^A)/\partial x_\lambda|\alpha\rangle|^2, \quad (6)$$

proving the theorem.

When $W=M_\alpha$, k_0 vanishes and the proof of the theorem breaks down. It is in fact well known that in the "elastic" weak reaction $\nu+N\rightarrow l+N$, a conserved vector current will contribute strongly in the forward direction.[6] We assume henceforth that $W\neq M_\alpha$.

II. TESTS OF CVC

Since the antisymmetric tensor term $\epsilon_{\lambda\sigma\gamma\delta}k_{1\gamma}k_{2\delta}$ vanishes under the hypotheses of Theorem 1, the characteristic parity-violating effects in weak interactions can arise only from vector-axial vector interference. Consequently, *if the vector current is conserved, and if m_l may be neglected, all parity violating effects must vanish in the parallel configuration.* This makes possible new experimental tests of the hypothesis that the vector current in $\Delta S=0$ leptonic reactions is conserved (CVC). Whereas previous tests have dealt with $\langle\beta|\mathcal{J}_\lambda{}^V|\alpha\rangle$ for $W\approx M_\alpha$ and various values of $k^2=(p_2-p_1)^2$, the new tests will study $\langle\beta|\mathcal{J}_\lambda{}^V|\alpha\rangle$ for $W\neq M_\alpha$ and $k^2\approx0$.

Let us work in the lab frame, in which α is at rest. We assume that α is unpolarized. Then, if CVC is *false*, the two simplest types of parity violating term which may appear in the differential cross section, in the parallel configuration, are:

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[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[2] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960), and references listed there.
[3] When $m_l=0$, both k_1 and k_2 are null vectors. If the space components of two null vectors are parallel in one Lorentz frame, they are parallel in all Lorentz frames. Hence "parallel configuration" is an invariant concept.
[4] The four-vectors have an imaginary time component: $p=(\mathbf{p},p_4)=(\mathbf{p},ip_0)$. The quantity $p^\star$ is defined by $\mathbf{p}^\star=\mathbf{p}^*$, $p_4{}^\star=-p_4{}^*$, where * denotes complex conjugation.
[5] In the frame in which β is at rest, $k_0=(W^2-M_\alpha{}^2-k^2)/(2W)$. When $m_l=0$, k is a null vector, so if k_0 is nonvanishing in any Lorentz frame it is nonvanishing in all Lorentz frames.
[6] T. D. Lee and C. N. Yang, Phys. Rev. **126**, 2239 (1962).

(A) The vector triple-product terms

$$\mathbf{q}_i\cdot(\mathbf{q}_j\times\mathbf{q}_k)\,, \tag{7}$$

where $\mathbf{q}_i$, $\mathbf{q}_j$, and $\mathbf{q}_k$ are any three distinct momenta chosen from among the lepton momentum $\mathbf{k}_2$ and the momenta $\mathbf{q}_1,\cdots,\mathbf{q}_n$ of

$$\mathbf{q}_1,\cdots,\mathbf{q}_n \text{ of } \beta_1,\cdots,\beta_n;$$

(B) The vector·pseudovector terms

$$\mathbf{q}_i\cdot\boldsymbol{\sigma}\,, \tag{8}$$

where $\boldsymbol{\sigma}$ is the spin of a baryon in β and $\mathbf{q}_i$ is any momentum chosen from among $\mathbf{k}_2$ and $\mathbf{q}_1,\cdots,\mathbf{q}_n$.

Since, in the parallel configuration, $\mathbf{k}_1$ and $\mathbf{k}_2$ are proportional, and since $\mathbf{k}_1=\mathbf{k}_2+\mathbf{q}_1+\cdots+\mathbf{q}_n$, the system β must contain at least three particles if there are to be enough linearly independent vectors to construct a nonvanishing triple product. Consequently, the reaction with the lowest threshold which could show a triple product term is *two-pion production:*

$$\nu(\mathbf{k}_1)+\alpha(\mathbf{0})\to l(\mathbf{k}_2)+\alpha'(\mathbf{q}_1)+\pi(\mathbf{q}_2)+\pi(\mathbf{q}_3)\,. \tag{7a}$$

If CVC is valid, the laboratory differential cross section must contain no term $\mathbf{k}_2\cdot(\mathbf{q}_2\times\mathbf{q}_3)$. (There is only one linearly independent triple product in two pion production.) Note that to test CVC it is not necessary to observe the recoil nucleus α'; it is enough to know the initial neutrino direction and to observe the lepton and the two pions.

Because nucleon polarizations are hard to measure, lambda kaon production,

$$\nu(\mathbf{k}_1)+\alpha(\mathbf{0})\to l(\mathbf{k}_2)+\alpha'(\mathbf{q}_1)+\Lambda(\mathbf{q}_2)+K(\mathbf{q}_3)\,, \tag{8a}$$

is the reaction with the lowest threshold in which terms of type (B) could be detected in practice. The Λ, through its decay asymmetry, analyzes its own polarization. If CVC is valid, the terms $\boldsymbol{\sigma}_\Lambda\cdot\mathbf{k}_2$, $\boldsymbol{\sigma}_\Lambda\cdot\mathbf{q}_2$ and $\boldsymbol{\sigma}_\Lambda\cdot\mathbf{q}_3$ must not appear, while $\boldsymbol{\sigma}_\Lambda\cdot(\mathbf{k}_2\times\mathbf{q}_2)$, $\boldsymbol{\sigma}_\Lambda\cdot(\mathbf{k}_2\times\mathbf{q}_3)$, and $\boldsymbol{\sigma}_\Lambda\cdot(\mathbf{q}_2\times\mathbf{q}_3)$ are allowed. Similar tests of CVC may be constructed for reactions with thresholds higher than those of Eqs. (7a) and (8a).

The proposed tests of CVC are strictly valid only when m_l is neglected. However, we will see in the next section that the main lepton mass correction to the matrix element does not give rise to interference between a conserved vector current and the axial vector current. Consequently, the tests should be good in practice even if the lepton is a muon.

III. TESTS OF PCAC

A. Lepton Mass Neglected

Let us now accept the truth of CVC. Then, neglecting m_l, the matrix element in the parallel configuration depends only on $\langle\beta|\partial\mathcal{J}_\lambda^A/\partial x_\lambda|\alpha\rangle$. In an attempt to find a general explanation for the validity of the Goldberger-Treiman formula for pion decay, it has been postulated by Nambu, Gell-Mann, and others that $\mathcal{J}_\lambda^A$ is partially conserved.[2] We denote by PCAC the hypothesis that the covariant amplitudes contributing to $\langle\beta|\partial\mathcal{J}_\lambda^A/\partial x_\lambda|\alpha\rangle$ satisfy unsubtracted dispersion relations in the variable k^2 and that these dispersion relations, for $-M_\pi^2<k^2\lesssim M_\pi^2$ and for *all* values of the other invariants formed from four-momenta in α and β, are dominated by the one-pion pole. ($M_\pi=$ the pion mass; we are of course considering only the case where the quantum numbers of α and β permit a one-pion pole.) Let $k_{0\beta}$ be the value of k_0 in the rest frame of β. If $k_{0\beta}^2/M_\pi^2\gg1$, the extrapolation from the physical value, $k^2\approx0$, to the pole at $k^2=-M_\pi^2$ will have little effect on the spinors and kinematics, and we have the covariant relation

$$\begin{aligned}\langle\beta|\partial\mathcal{J}_\lambda^A/\partial x_\lambda|\alpha\rangle &= -ik_\lambda\langle\beta|\mathcal{J}_\lambda^A|\alpha\rangle=(2k_0^{1/2})\mathcal{T}(\pi^++\alpha\to\beta)(k^2+M_\pi^2)^{-1}\\ &\quad\times(2k_0)^{1/2}\langle\pi^+|\partial\mathcal{T}_\lambda^A/\partial x_\lambda|0\rangle.\end{aligned} \tag{9}$$

Here $\mathcal{T}(\pi^++\alpha\to\beta)$ is the transition amplitude for $\pi^++\alpha\to\beta$, with the incident π^+ of energy k_0 and with the momentum of the incident π^+ parallel to $\mathbf{k}$. The Goldberger-Treiman relation, itself a consequence of PCAC, may be used to express the pion-decay matrix element in terms of g_A, the beta decay axial-vector coupling constant:

$$(2k_0)^{1/2}\langle\pi^+|\partial\mathcal{J}_\lambda^A/\partial x_\lambda|0\rangle=-i2^{1/2}M_Ng_Ag_r^{-1}M_\pi^2. \tag{10}$$

Numerically, $g_A\approx1.2\times10^{-5}M_N^{-2}$; M_N is the nucleon mass and g_r is the rationalized, renormalized pion-nucleon coupling constant ($g_r^2/4\pi\approx14$). Combining Eqs. (9) and (10) gives

$$\begin{aligned}k_\lambda\langle\beta|\mathcal{J}_\lambda^A|\alpha\rangle &=(2k_0)^{1/2}\mathcal{T}(\pi^++\alpha\to\beta)2^{1/2}M_Ng_Ag_r^{-1}M_\pi^2(k^2+M_\pi^2)^{-1}\\ &=(2k_0)^{1/2}\mathcal{T}(\pi^++\alpha\to\beta)2^{1/2}M_Ng_Ag_r^{-1}\\ &\quad\times[1-k^2(k^2+M_\pi^2)^{-1}].\end{aligned} \tag{11}$$

This equation may be used to express the weak-reaction cross section in the parallel configuration in terms of the cross section for $\pi^++\alpha\to\beta$. Before carrying through the details we will consider lepton mass corrections.

B. Lepton Mass Corrections

Up to this point we have neglected m_l. Now let us compute the principal lepton mass corrections. We will find that the lepton mass corrections, while not contributing significantly to terms of the form (vector)·(axial vector) in the squared matrix element, make an

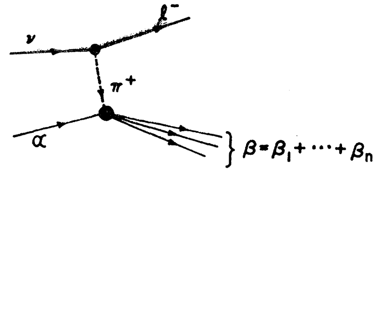

FIG. 1. Diagram giving rise to $\langle\beta|\mathcal{J}_\lambda^{A\mathrm{II}}|\alpha\rangle$.

important contribution to terms of the form (axial vector)·(axial vector).

Consider the diagram shown in Fig. 1. We denote by $\langle\beta|\mathcal{J}_\lambda^{AII}|\alpha\rangle$ the contribution of this diagram to $\langle\beta|\mathcal{J}_\lambda^{A}|\alpha\rangle$, and by $\langle\beta|\mathcal{J}_\lambda^{AI}|\alpha\rangle$ everything that is left over. It is easy to see that

$$\langle\beta|\mathcal{J}_\lambda^{AII}|\alpha\rangle=(2k_0)^{1/2}\mathcal{T}(\pi^++\alpha\to\beta)2^{1/2}M_Ng_Ag_r^{-1}\times(-k_\lambda)(k^2+M_\pi^2)^{-1}, \quad (12)$$

from which it follows that

$$k_\lambda\langle\beta|\mathcal{J}_\lambda^{AI}|\alpha\rangle=(2k_0)^{1/2}\mathcal{T}(\pi^++\alpha\to\beta)2^{1/2}M_Ng_Ag_r^{-1}. \quad (13)$$

Although $\langle\beta|\mathcal{J}_\lambda^{AII}|\alpha\rangle$ is a lepton mass correction, it must be retained because $m_l^2(k^2+M_\pi^2)^{-1}$ is of order unity when l is a muon. Other lepton mass corrections involve large masses in the denominator and may reasonably be neglected. Keeping terms of first order in m_l^2 in the kinematics gives[7]

$$\begin{aligned}&k_1=k_{10}k_0^{-1}k+bp_1,\quad k_2=k_{20}k_0^{-1}k+bp_1;\\&2k\cdot p_1b=2k_1\cdot k_2=-m_l^2k_{10}k_{20}^{-1},\quad k^2=m_l^2k_0k_{20}^{-1}.\end{aligned} \quad (14)$$

If the vector current is conserved, it follows that

$$\begin{aligned}\langle\beta|\mathcal{J}_\lambda^V+\mathcal{J}_\lambda^A|\alpha\rangle&\langle\beta|\mathcal{J}_\sigma^V+\mathcal{J}_\sigma^A|\alpha\rangle^\star T_{\lambda\sigma}\\&=2k_{10}k_{20}k_0^{-2}|\langle\beta|k\cdot\mathcal{J}^A|\alpha\rangle|^2\\&+2b(k_{10}+k_{20})k_0^{-1}\,\mathrm{Re}[\langle\beta|k\cdot\mathcal{J}^A|\alpha\rangle\langle\beta|p_1\cdot\mathcal{J}^{AII}|\alpha\rangle^*]\\&+2b^2|\langle\beta|p_1\cdot\mathcal{J}^{AII}|\alpha\rangle|^2\\&+\tfrac{1}{2}m_l^2k_{10}k_{20}^{-1}\{\langle\beta|\mathcal{J}_\lambda^{AII}|\alpha\rangle\langle\beta|\mathcal{J}_\lambda^{AII}|\alpha\rangle^\star\\&\qquad+2\,\mathrm{Re}[\langle\beta|\mathcal{J}_\lambda^{AI}|\alpha\rangle\langle\beta|\mathcal{J}_\lambda^{AII}|\alpha\rangle^\star]\}.\end{aligned} \quad (15)$$

We have retained m_l^2 only in terms where there is one factor $(k^2+M_\pi^2)^{-1}$ for each factor m_l^2. *No vector–axial-vector interference terms are of this form.* Substituting Eqs. (11) through (14) into Eq. (15) and performing algebraic simplification leads to the following theorem:

Theorem 2. Suppose that CVC and PCAC are true. Consider the parallel configuration in $\nu+\alpha\to l^-+\beta$, for $k_{0\beta}$ satisfying $k_{0\beta}^2/M_\pi^2\gg1$. Let m_l^2 be retained only where it occurs in the combination $m_l^2(k^2+M_\pi^2)^{-1}$. Then the invariant matrix element $\mathfrak{M}$ for $\nu+\alpha\to l^-+\beta$, squared and averaged over lepton spin, is related to the invariant matrix element[8] $\mathfrak{M}(\pi^++\alpha\to\beta)$ for $\pi^++\alpha\to\beta$, with the π^+ of energy k_0 and with the π^+ momentum parallel to $\mathbf{k}$, by

$$\langle|\mathfrak{M}|^2\rangle=\frac{4M_N^2}{g_r^2}\frac{k_{10}k_{20}}{k_0^2}g_A^2\times\left[1-\frac{m_l^2k_0}{2(M_\pi^2k_{20}+m_l^2k_0)}\right]^2|\mathfrak{M}(\pi^++\alpha\to\beta)|^2. \quad (16)$$

In computing k_{10}, k_{20}, and k_0 in Eq. (16), the lepton mass should be neglected. Then Eq. (16) will be formally covariant, since ratios of the time components of parallel null vectors, such as k_{10}/k_0 and k_{20}/k_0, are invariant quantities.

Corollary 1. Under the hypotheses of the theorem, the energy, angle, and polarization distributions of the particles in β, in the reaction $\nu+\alpha\to l^-+\beta$, will be identical with the distributions in the reaction $\pi^++\alpha\to\beta$ (for the same invariant mass W of β in the two processes).

Corollary 2. Under the hypotheses of the theorem, the lepton differential cross section $d\sigma/d\Omega_l$ of $\nu+\alpha\to l^-+\beta$, in the laboratory frame (the rest frame of α), is given by

$$\frac{d\sigma}{d\Omega_l}=\int\frac{dW}{k_0^2}\left(\frac{W}{M_\alpha}\right)^2(k_0^2-M_\pi^2)^{1/2}\left(\frac{M_\pi}{F}\right)^2\frac{k_{20}^2}{4\pi^3}g_A^2\times\left[1-\frac{m_l^2k_0}{2(M_\pi^2k_{20}+m_l^2k_0)}\right]^2\sigma(W), \quad (17)$$

where $\sigma(W)$ is the total cross section at total energy W in the β rest frame, for $\pi^++\alpha\to\beta$. Also,

$$k_0=(W^2-M_\alpha^2+M_\pi^2)/(2W),$$
$$k_{20}=(M_\alpha^2+2M_\alpha E-W^2)/(2W),$$

E is the neutrino energy in the laboratory frame, and $F=M_\pi g_r/(2M_N)\approx1.0$. The formulas for $\bar\nu+\alpha\to l^++\beta$ corresponding to those given above are obtained by replacing π^+ by π^-.

Use of Corollary 1 to test PCAC does not require knowledge of the neutrino energy spectrum, since for a given W the energy, angle and polarization distributions of the particles in β are independent of the neutrino energy E. Testing PCAC by making a quantitative comparison of $d\sigma/d\Omega_l$ with Eq. (17) of Corollary 2 does require a knowledge of the neutrino spectrum. Since all dependence on E in Eq. (17) is contained in the factors $k_{20}^2[1-\frac{1}{2}m_l^2k_0(M_\pi^2k_{20}+m_l^2k_0)^{-1}]^2$, the weighting over the spectrum is easy to carry out once the spectrum is known.

IV. EXTRAPOLATION IN k^2

Theorem 2 requires that $k_{0\beta}^2/M_\pi^2$ be much larger than unity. This condition is necessary for it to be legitimate to extrapolate from $k^2\approx m_l^2k_0k_{20}^{-1}$ to $k^2=-M_\pi^2$ in the kinematics of the reaction $\nu+\alpha\to l^-+\beta$. Since $k_{0\beta}\approx(W-M_\alpha)(W+M_\alpha)/(2W)$, the condition

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[7] Actually, $k_1=k_{10}k_0^{-1}k+ak+bp_1$, $k_2=k_{20}k_0^{-1}k+ak+bp_1$, where a is of first order in m_l^2. We have dropped the term ak in Eq. (14) because it does not lead to important lepton mass corrections.

[8] The transition amplitude $\mathcal{T}(\pi^++\alpha\to\beta)$ and the invariant matrix element $\mathfrak{M}(\pi^++\alpha\to\beta)$ are related by

$$\mathcal{T}(\pi^++\alpha\to\beta)=\prod_{i,j}\left(\frac{m_i}{p_{i0}}\frac{1}{2p_{j0}}\right)^{1/2}\mathfrak{M}(\pi^++\alpha\to\beta).$$

The factor of proportionality is just the product of the normalization factors for the wave functions of π^+, α and of all the particles in β. The S matrix is given in terms of $\mathcal{T}$ by

$$S_{fi}=\delta_{fi}+(2\pi)^4i\delta(p_f-p_i)\mathcal{T}.$$

$k_{0\beta}^2/M_\pi^2 \gg 1$ will be satisfied as long as $W-M_\alpha \gtrsim 4M_\pi$. Thus, for most weak multiparticle production reactions, Theorem 2 is valid as it stands.

However, in the interesting case of single-pion production in the (3,3) resonance region, $W-M_\alpha < 4M_\pi$ and Theorem 2 must be modified. This is done by replacing $\mathfrak{M}(\pi^+ + \alpha \to \pi + \alpha')$ by $\mathfrak{M}^c(\pi^+ + \alpha \to \pi + \alpha')$, where $\mathfrak{M}^c$ is the invariant matrix element computed from the covariant amplitudes for $\pi^+ + \alpha \to \pi + \alpha'$ by using the correct kinematics, with $k^2 \approx m_l^2 k_0 k_{20}^{-1}$, for the reaction $\nu + \alpha \to l^- + \pi + \alpha'$. When α is a single nucleon N,

$$\mathfrak{M}(\pi^+ + N \to \pi + N') = (4\pi W/M_N)\chi_f^\dagger [f_1(W, \hat{q}\cdot\hat{k}) + \boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\sigma}\cdot\hat{k} f_2(W,\hat{q}\cdot\hat{k})]\chi_i, \quad (18)$$

with f_1 and f_2 the usual center-of-mass pion-nucleon scattering amplitudes,[9] and with $\hat{q}$ and $\hat{k}$ unit vectors, in the center of mass, along the momenta of the final and initial pion, respectively. Calculation of $\mathfrak{M}^c$ shows that[10]

$$\mathfrak{M}^c(\pi^+ + N \to \pi + N') = (4\pi W/M_N)\chi_f^\dagger [g_1(W, \hat{q}\cdot\hat{k}) + \boldsymbol{\sigma}\cdot\hat{q}\boldsymbol{\sigma}\cdot\hat{k} g_2(W,\hat{q}\cdot\hat{k})]\chi_i, \quad (19)$$

where

$$g_1(W,\hat{q}\cdot\hat{k}) \approx f_1(W,x),$$
$$g_2(W,\hat{q}\cdot\hat{k}) \approx [k_0/(k_0^2 - M_\pi^2)^{1/2}] f_2(W,x), \quad (20)$$

$$x = [k_0/(k_0^2-M_\pi^2)^{1/2}]\hat{q}\cdot\hat{k} + [k_0(M_\pi^2+k^2)/2W(k_0^2-M_\pi^2)],$$

and where $k_0 = (W^2 - M_N^2 + M_\pi^2)/(2W)$. Clearly, when $k_0^2/M_\pi^2 \gg 1$ one finds that $g_{1,2}(W,\hat{q}\cdot\hat{k}) \approx f_{1,2}(W,\hat{q}\cdot\hat{k})$, as is expected.

If only the dominant (3,3) partial wave is retained, the main effect of Eq. (20) is to replace $\sigma_{3,3}(W)$ in Corollary 2 by $\sigma_{3,3}(W)k_0^2/(k_0^2-M_\pi^2)$. If, in additino, the lepton mass and nucleon recoil effects are neglected, Eq. (17) reduces to the result obtained from the static model by Bell and Berman.[11] This agreement with the static model is not surprising. The (3,3) projections of the Born terms for weak pion production and for pion-nucleon scattering can be shown to satisfy the PCAC proportionality. In the static model, the entire matrix element is determined by the (3,3) projection of the Born term and by the experimental (3,3) resonance parameters. Hence, in the static model, the weak pion production and pion-nucleon scattering matrix elements satisfy the PCAC proportionality.

Note added in proof. The considerations of this paper also apply to the decays $\Sigma^\pm \to \Lambda + e^\pm + (\nu/\bar{\nu})$, when the electron is relativistic and emerges parallel to the neutrino. For example, if CVC and PCAC are true, measurement of the differential decay rate in the parallel configuration would determine the strong $\Sigma\Lambda\Pi$ coupling constant.

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[9] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957). In Eqs. (18) through (20), isotopic spin indices have been suppressed.

[10] The calculation is done as follows: We are considering the reaction $\nu(k_1)+N(p_1) \to l(k_2)+N'(s)+\pi(q)$. Let us define the variables ν and ν_B by $\nu = -\frac{1}{2}(p_1+s)\cdot q/M_N$ and $\nu_B = \frac{1}{2}q\cdot k/M_N$. The corrected matrix element $\mathfrak{M}^c$ is given by

$$\mathfrak{M}^c = \bar{u}(s)[A^{\pi N}(\nu,\nu_B) - i\gamma\cdot q B^{\pi N}(\nu,\nu_B)]u(p_1),$$

where $A^{\pi N}(\nu,\nu_B)$ and $B^{\pi N}(\nu,\nu_B)$ are the covariant amplitudes for pion-nucleon scattering. [In pion-nucleon scattering, $\pi(q_1) + N(p_1) \to \pi(q) + N'(s)$, the variables ν and ν_B are defined by $\nu = -\frac{1}{2}(p_1+s)\cdot q/M_N$ and $\nu_B = \frac{1}{2}q\cdot q_1/M_N$.] Expressing $\mathfrak{M}^c$ in terms of pion-nucleon center of mass variables, using the kinematics appropriate to weak pion production, leads to Eq. (19).

[11] J. S. Bell and S. M. Berman, Nuovo Cimento **25**, 404 (1962). Bell and Berman neglect all lepton mass effects.